

$\mathcal{N} = 2$ GAUGE-GRAVITY DUALS

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We study the $\mathcal{N} = 2$ analog of the Klebanov-Strassler system. We first review the resolution of singularities by the enhançon mechanism, and the physics of fractional branes on an orbifold. We then describe the exact $\mathcal{N} = 2$ solution. This exhibits a duality cascade as in the $\mathcal{N} = 1$ case, but the singularity resolution is the characteristic $\mathcal{N} = 2$ enhançon. We discuss some related solutions and open issues.

1. Introduction

Maldacena's duality¹ between IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is a remarkable relation between string theory and quantum field theory, and it is important to extend it to situations of less supersymmetry. As with all dualities, information flows in both directions. In one direction, the less supersymmetric gauge/gravity duals give quantitative solutions to strongly coupled gauge theories with confinement and chiral symmetry breaking. In the other, they give information about string compactifications with realistic amounts of supersymmetry as well as large warp factors.

In this talk I will focus on $D = 4$, $\mathcal{N} = 2$ duals. Systems with $\mathcal{N} = 2$ supersymmetry are often studied as a step towards $\mathcal{N} = 1$, taking advantage of the extra supersymmetry. Happily, recent progress in $\mathcal{N} = 1$ gauge/gravity duals has been quite rapid, so this motivation is no longer as great. Nevertheless, it is interesting to study the $\mathcal{N} = 2$ systems, both for the perspective that they give on $\mathcal{N} = 1$ and because there are some interesting open questions.

The supersymmetry and conformal invariance can be broken in various ways. I will focus on breaking by the combination of orbifolding and fractional brane fluxes, but will also discuss briefly breaking by small perturbations (the latter is based on work with Buchel and Peet²). A central issue is that the breaking of the conformal invariance of the AdS space often leads to the horizon being replaced by a naked singularity. It has now been seen in many examples^{3,4,5,6} that the singularity is not actually present, though the mechanism that eliminates it varies from case to case. There is a particular phenomenon, the enhançon, that is characteristic of $\mathcal{N} = 2$ systems.³

In section 2 I review singularity resolution by the enhançon. In section 3 I review gauge/gravity duality with orbifolding and fractional branes, and describe an $\mathcal{N} = 2$ analog of Seiberg duality. In section 4 I study the $\mathcal{N} = 2$ version of

the Klebanov-Strassler solution,⁵ showing that there is a duality cascade in the UV as in the $\mathcal{N} = 1$ case, but that the infrared behavior is the characteristic $\mathcal{N} = 2$ enhançon. Finally, I discuss dualities relating various solutions, and some open issues.*

2. The Enhançon

For convenience let us focus on pure $\mathcal{N} = 2$ $SU(N)$ Yang-Mills theory. The Coulomb branch is N complex dimensional. Correspondingly the brane system is made up of N constituents, each moving in a two-dimensional transverse space: these are 3-branes in six dimensions, with four dimensions having been compactified or reduced.

When the constituents are well-separated, the metric g_{ij} on the moduli space of their positions is flat, but at sufficiently small separations it becomes negative. This is unphysical: the condition

$$g_{ij} > 0 \tag{1}$$

thus excludes a region in the interior of the moduli space. On the boundary of this region the metric has a zero, so at least one of the 3-branes has vanishing tension. A 3-brane of vanishing tension is of course a rather special object, and so it must be at a spacetime point of nontrivial infrared physics. In the $2 + 1$ dimensional case it is a point of enhanced gauge symmetry, hence the name *enhançon*³ for such point; in the $3 + 1$ dimensional case there are tensionless strings. We will also use the term *enhançon* for the codimension one surface in moduli space where the metric has a zero.

Generically the metric has only one zero, but there are points in the N -dimensional moduli space where multiple 3-branes are tensionless. If we try to bring the 3-branes as close together as possible, they will all be tensionless, and their positions will map out some closed curve in spacetime — the enhançon points fill out, in the large- N limit, a closed curve.

In refs. 3 and 9 the supersymmetry was broken to $\mathcal{N} = 2$ by different means, but the IR physics in both is governed by the enhançon. In ref. 3 the constituents were D7-branes wrapped on K3, with $\mathcal{N} = 4$ broken to $\mathcal{N} = 2$ by curvature. For simplicity it was assumed that these were distributed on a ring; as its radius was reduced they all became tensionless simultaneously. In ref. 9 the breaking to $\mathcal{N} = 2$ was by a perturbation of the $AdS_5 \times S^5$ background, corresponding¹⁰ to a mass perturbation of the $\mathcal{N} = 4$ theory. A family of solutions were obtained by the lift of five-dimensional solutions, correspond to a one-parameter ($\gamma < 0$) curve in moduli space. At the limiting value $\gamma = 0$ the constituent D3-branes all lay on enhançon points, which in this case formed a segment of the real axis.²

*I should point out that Igor Klebanov, with a series of collaborators, has pioneered the study of this fascinating system. Much of my talk is a review of his work, with a few new ingredients — the exact $\mathcal{N} = 2$ solution, the analysis of the $\mathcal{N} = 2$ duality cascade and singularity resolution, and some of the final remarks. See also the talks by Klebanov and Strassler.

After this was written I learned of the recent ref. 7, with which it has substantial overlap. Ref. 8, which has since appeared, presents a new $\mathcal{N} = 2$ supergravity dual; how it is related to the solutions described in section 5.2 is not yet clear.

The enhançon has a simple gauge theory interpretation.^{3,2,11} In the $\mathcal{N} = 2$ theory the metric on moduli space receives corrections only at tree level, one loop, and nonperturbatively. The whole is positive definite, but the perturbative part alone goes negative in parts of moduli space.¹² The instanton corrections are of order $e^{-O(g_{\text{YM}}^{-2})}$. Since $g_{\text{YM}}^2 = O(N^{-1})$, the perturbative part is accurate until one reaches a point on moduli space where $g_{\text{YM}}^{-2} \rightarrow 0$, at which point the corrections must suddenly become important. Noting that the metric on moduli space is essentially g_{YM}^{-2} , the enhançon surface in moduli space is precisely where the instanton effects become important. Of course, the moduli space does not end there. By holomorphy it must continue into the interior, but as argued in ref. 3 it can no longer be interpreted as the moduli space of the positions of pointlike constituents: naive continuation of the supergravity solution into the interior would be singular. Rather, the moduli represent the internal state of the theory at the enhançon; it would be interesting to make this more precise.

In other words, the enhançon is the large- N manifestation of the Seiberg-Witten curve,¹² which takes the form¹³

$$y^2 = x^{2N} - \phi^{2N} + \Lambda^{2N} \quad (2)$$

for a \mathbf{Z}_N -invariant Higgs v.e.v. of magnitude ϕ . For $\phi > \Lambda$, the Λ^{2N} term is negligible and the physics is perturbative; for $\phi < \Lambda$ the instanton correction Λ^{2N} is dominant. Note that for $\phi = \Lambda$ the curve degenerates and one has an Argyres-Douglas point; it would be interesting to see whether this has any special geometric interpretation in the enhançon context.^a

3. Orbifolds and Fractional Branes

3.1. D3-Branes on the \mathbf{Z}_2 orbifold

The symmetries of $AdS_5 \times S^5$ can be broken by a perturbation which becomes linear at the boundary, corresponding to a relevant perturbation of the gauge theory Hamiltonian,¹⁰ or by a large deformation at the boundary. Generally the latter corresponds to placing the D3-branes at a singular point of the transverse space.^{14,15,16,17,18} I will consider the simplest case, of a \mathbf{Z}_2 orbifold

$$R: \quad x^{4,5} \rightarrow x^{4,5}, \quad x^{6,7,8,9} \rightarrow -x^{6,7,8,9}. \quad (3)$$

This breaks half of the supersymmetry, leaving $\mathcal{N} = 2$. In this section I will review the gauge/gravity duality for this orbifold.^{14,19}

On the string side one restricts as usual to \mathbf{Z}_2 -invariant configurations and adds in twisted sectors, states localized on the fixed plane $x^{6,7,8,9} = 0$. On the gauge side, the massless open string spectrum is obtained by standard technology.^{20,21} To describe N D3-branes on the orbifolded space we also need their N images (fig. 1a), giving a total of $2N$ Chan-Paton degrees of freedom labeled $i \in 1, \dots, 2N$. The orbifolding acts both on the oscillator state ψ of an open string and on its

^aThis remark is due to Cumrun Vafa.

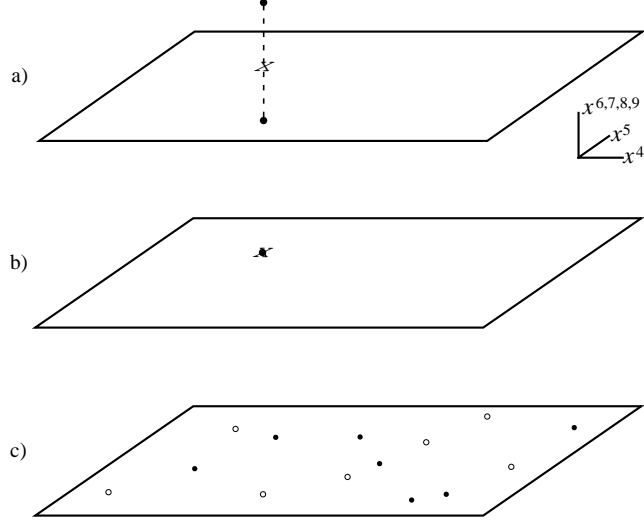


Fig. 1. a) N D3-branes above the fixed plane, and their images below. b) All $2N$ branes+images coincident on the plane. c) N D5-branes (closed circles) and N anti-D5-branes (open circles) separated on the fixed plane.

Chan-Paton degrees of freedom:

$$R|i, j, \psi\rangle = \gamma_{ii'} \gamma_{jj'} |i', j', \hat{R}\psi\rangle, \quad (4)$$

where the matrix γ interchanges each D3-brane with its image,

$$\gamma = \begin{bmatrix} 0 & I_N \\ I_N & 0 \end{bmatrix}. \quad (5)$$

The orbifold projection retains states even under R .

We are interested in the case that the D3-branes (and so their images) lie in the fixed plane, as in fig. 1b. To analyze this case it is useful to go to a different basis for the Chan-Paton factors,

$$\gamma = \begin{bmatrix} I_N & 0 \\ 0 & -I_N \end{bmatrix}. \quad (6)$$

In this basis, states in the diagonal blocks survive the projection if their oscillator state is even under \hat{R} and states in the off-diagonal blocks survive if it is odd. This leaves the massless states

$$\begin{bmatrix} A^\mu, X^{4,5} & X^{6,7,8,9} \\ X^{6,7,8,9} & A^\mu, X^{4,5} \end{bmatrix} \rightarrow \begin{bmatrix} A^\mu, \phi & A_{1,2} \\ B_{1,2} & \tilde{A}^\mu, \tilde{\phi} \end{bmatrix}. \quad (7)$$

In the second form we have collected the scalars into complex pairs. The gauge group is $U(N) \times U(N)$, with an adjoint $\mathcal{N} = 2$ vector multiplet and two $(\mathbf{N}, \bar{\mathbf{N}}) + (\bar{\mathbf{N}}, \mathbf{N})$ hypermultiplets. The notation (7) corresponds to $\mathcal{N} = 1$ multiplets.

The overall $U(1)$ decouples from the bulk dynamics, so the orbifolded string theory is the dual to the $\mathcal{N} = 2$ $SU(N) \times SU(N) \times U(1)$ gauge theory with two bifundamental hypermultiplets. Not surprisingly, since there is still an AdS_5 factor, this is a conformally invariant theory.^{14,22,b}

3.2. Fractional branes

The system becomes more interesting when generalized as follows. In the basis (6), we can generalize to blocks of different size,

$$\gamma = \begin{bmatrix} I_{N+M} & 0 \\ 0 & -I_N \end{bmatrix}. \quad (8)$$

This corresponds to a $U(N+M) \times U(N)$ gauge theory with two bifundamental hypermultiplets. This γ can no longer be brought to the form (5). A $2N$ -dimensional submatrix can be, and so N D3-branes can be moved off the fixed plane, but this leaves M ‘half-D3-branes.’ The ‘half’ is because these do not have images, and so for example couple with half the strength of a D3-brane to closed string states.

We can also form these half-D3-branes directly in the $U(N) \times U(N)$ system. The vevs of ϕ and $\tilde{\phi}$ give $2N$ independent coordinates in the fixed plane, corresponding to the N D3-branes separating into $2N$ half-D3-branes. These are of two types (fig. 1c), according to whether γ acts as $+1$ or -1 on the given Chan-Paton index.

These half-branes have a simple interpretation.²⁴ There are five massless scalar twisted states in the IIB theory. Three correspond to blowups of the fixed point $x^{6,7,8,9} = 0$. Hidden in the fixed point is a zero-size S^2 , which acquires a finite size when the fixed point is blown up into a smooth space. The other two scalars are θ -parameters from the NS-NS and R-R two-form potentials integrated over the S^2 ,

$$\theta_B = \frac{1}{2\pi\alpha'} \int_{S^2} B_2, \quad \theta_C = \frac{1}{2\pi\alpha'} \int_{S^2} C_2. \quad (9)$$

c The two types of half-brane correspond to D5-branes or anti-D5-branes wrapped on this S^2 . Being D-branes, these should have a (magnetic) coupling to θ_C . This coupling is given by a disk amplitude with a single vertex operator. The vertex operator has a branch cut extending to the boundary, and includes a factor of γ acting on the boundary state. Thus the coupling is proportional to the trace of γ , and so is zero for a full D3-brane, but ± 1 for the two types of half-brane.

The twisted state coupling measures the D5 charge. The half unit of D3-brane charge can be understood as follows. A D5-brane couples to the Ramond background as

$$\int (C_6 + B_2 \wedge C_4). \quad (10)$$

The first term is its magnetic coupling to C_2 . The second is a D3 charge equal to $\theta_B/2\pi$, and the free orbifold CFT is known²⁵ to corresponds to $\theta_B = \pi$; indeed, this

^bTo be precise, the $U(1)$ is not conformal but IR free.²³ How this is manifests on supergravity side is a puzzle, because of the AdS_5 factor.

is probably the most physical way to derive that fact. Thus, fig. 1c corresponds to N D3-branes separating into N $[\text{D5} + \frac{1}{2}\text{D3}]$ plus N $[\overline{\text{D5}} + \frac{1}{2}\text{D3}]$. The more general matrix γ corresponds to

$$(N + M) \left[\text{D5} + \frac{1}{2}\text{D3} \right] \oplus N \left[\overline{\text{D5}} + \frac{1}{2}\text{D3} \right] \quad (11)$$

Thus, the $U(N + M)$ gauge factor is associated with the wrapped D5-branes, and the $U(N)$ factor with the wrapped anti-D5-branes.

3.3. An $\mathcal{N} = 2$ Seiberg duality

The final issue to understand is the effect of varying θ_B , particularly on the couplings of the two gauge groups. First let us stay in the range $0 \leq \theta_B \leq 2\pi$. The system (11) evolves to

$$(N + M) \left[\text{D5} + \frac{1}{2\pi}\theta_B \text{D3} \right] \oplus N \left[\overline{\text{D5}} + (1 - \frac{1}{2\pi}\theta_B) \text{D3} \right]. \quad (12)$$

For these objects, the tension is simply proportional to the magnitude of the D3-brane charge,

$$\tau = \tau_3 |Q_3|, \quad \tau_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s}, \quad (13)$$

and so is respectively $\frac{1}{2\pi}\theta_B \tau_3$ and $(1 - \frac{1}{2\pi}\theta_B) \tau_3$ for the constituents (12). The D5-brane tension makes no contribution because the sphere on which the D5-brane is wrapped has zero size. Note that — as long as all constituents have the same sign of Q_3 — the full system satisfies the BPS property (13) and so is a BPS state, in spite of having both D5-branes and anti-D5-branes. Of course, if the S^2 is blown up to finite size, the BPS mass will have term involving $|Q_5|$ and the fully separated system of fig. 1c will no longer be supersymmetric — the fractional branes of opposite Q_5 will bind into full D3-branes.

The gauge field action comes from expanding out the Born-Infeld Lagrangian

$$-\tau \sqrt{-\det(G_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}. \quad (14)$$

It follows that the gauge coupling is just proportional to the tension, and so for the constituents (12)²²

$$\frac{4\pi}{g_{SU(N+M)}^2} = \frac{\theta_B}{2\pi g_s}, \quad \frac{4\pi}{g_{SU(N)}^2} = \frac{2\pi - \theta_B}{2\pi g_s}. \quad (15)$$

The angle θ_B is a periodic variable, but the system that we are studying undergoes an interesting spectral flow. Consider increasing θ_B past 2π . The D3 charge of the anti-D-branes becomes negative, so the system (12) is no longer BPS. However, it can remain BPS by rearranging into new constituents as θ_B goes through 2π ,

$$(N + 2M) \left[\text{D5} + (\frac{1}{2\pi}\theta_B - 1) \text{D3} \right] \oplus (N + M) \left[\overline{\text{D5}} + (2 - \frac{1}{2\pi}\theta_B) \text{D3} \right]. \quad (16)$$

Thus the gauge group changes as well, from $U(N + M) \times U(N)$ to $U(N + 2M) \times U(N + M)$. This is the same Seiberg duality²⁶ that appears in $\mathcal{N} = 1$.⁵ Note that we are treating θ_B as a constant, but the gauge couplings will run with energy; we will take account of this in the next section.

For $M = 0$ there is no spectral flow and θ_B is simply a periodic variable; it is remarkable that the inverse gauge coupling is thus also a periodic variable,²⁷ which can be smoothly taken to 0 and then back to positive values. For $M \neq 0$ it would be interesting to understand better the nature of the flow. Note that there is a $U(N + M)$ factor on both sides, whose gauge coupling is finite and continuous through the transition. Note also that we can think of $U(N + 2M)$ as related to $U(N)$ through Higgsing by the scalar field ϕ in the vector multiplet, with the $U(N + M)$ group as a spectator.^c However, this simple picture is probably not correct: note that the $U(N + M)$ factor initially acts on the D5-brane Chan-Paton factors, but after the transition it acts on the anti-D5-brane Chan-Paton factors; simple Higgsing would not have this effect. Also, we might expect some form of electric-magnetic duality to relate the two sides;²⁶ it is not clear how to see this.

4. The Supergravity Dual

We now consider the supergravity solution corresponding to a distribution of D5- and D3-branes. We first consider the case that the D5-branes can be treated as a perturbation, as in ref. 19. With the three-form flux having components only transverse components, its equation of motion takes the simple form

$$d[Z^{-1}(*_6 G_3 - iG_3)] = 0 , \quad (17)$$

where Z is the harmonic (with sources) function appearing in the D3-brane background and $G_3 = F_3 - \tau H_3$. In the present case, $G_3 = dA_2$ with

$$A_2 = \theta(x^4, x^5)\omega_2 , \quad (18)$$

where $\theta = \theta_C - \tau\theta_B$ and ω_2 is the two-form localized at the fixed point, with unit integral over the small two-sphere. In the orbifold limit ω_2 has delta function support, but expressions can be regularized by slightly blowing up the fixed point. The equation of motion then takes the form

$$\partial_z(Z^{-1}\partial_{\bar{z}}\theta) = 0 \quad (19)$$

in terms of $z = x^4 + ix^5$.

A simple class of solutions is that θ be analytic, and this moreover is the condition for $\mathcal{N} = 2$ supersymmetry. The wrapped D5-branes are magnetic sources for

^cThe $U(N+2M) \times U(N+M)$ can also be broken to $U(N+M) \times U(N) \times U(1)$'s by a combination of ϕ and $\tilde{\phi}$, or by the hypermultiplets A_i and B_i . The former most closely reflects the actual distribution of D-brane charges, but it is unlikely that the rearrangement between constituents (12) and (16) can be understood in terms of classical Higgsing. I would like to thank Ofer Aharony for extensive discussions of this point.

θ_C , so^d

$$\theta = -2i \sum_i q_i \ln(z - z_i) , \quad (20)$$

where $q_i = \pm 1$ and z_i are the D5 charge and position of each wrapped 5-brane. The branch cut in the logarithm corresponds to the equivalence $\theta_C \cong \theta_C + 2\pi$, with a factor of 2 arising because this is the self-intersection number of the S^2 . This is all as in ref. 19. The one new ingredient I have to add is the exact solution with backreaction. With θ analytic, the solution is in the class found in ref. 28: the three-form flux acts precisely as a D3-brane source in the other field equations, so there backreaction appears in a modified harmonic function Z' . That is, in addition to a source from any explicit D3-branes, there is a source on the fixed plane, proportional to $|\partial_z \theta|^2$. For example, the source for dF_5 is proportional to $H_3 \wedge F_3$.

Consider now M D5-branes at the origin, so that

$$\theta = -2iM \ln z . \quad (21)$$

The first point to notice is that θ_B is a function of radius, $gM \ln r$. Through eq. (15) this translates into a running of the couplings, which agrees with the $\mathcal{N} = 2$ beta function.¹⁹ Thus there is a duality cascade, with the successive duality steps separated in scale by a factor $e^{2\pi/gM}$; note that the exponent is small in the supergravity regime.

Now consider the backreaction. The effective D3 charge density is proportional to $|\partial_z \theta|^2 = M^2 |z|^{-2}$ and so logarithmically divergent both in the UV and IR. The normalization is such that the total D3 charge between two duality cascades is exactly M . This is as expected: the size of the gauge group matches the D3 charge, but as in the Klebanov-Strassler (KS) solution⁵ these D3-branes are realized as flux, not mobile D-objects (more on this in the next section).

The UV divergence is as in the KS solution, reflecting the unbounded growth of the gauge group. One difference is that in the $\mathcal{N} = 1$ KS case, there is no known way to terminate the cascade in a four-dimensional field theory, while in the $\mathcal{N} = 2$ case it is possible: the $U(N+M) \times U(N)$ theory can be obtained as the low energy limit of a spontaneously broken $U(N+M) \times U(N+M)$ theory, which is conformal in the UV. In terms of branes, beginning with $N+M$ D3-branes we can pull M anti-D5-branes out to long distance.

The IR divergence, however, is a problem. It can be ‘renormalized,’ leaving a finite backreaction, by adding a negative infinite D3 charge at the origin. However, this solution is singular. For a sufficiently small r the total D3 charge within is negative, and so the warp factor Z' changes sign. Where it goes to zero, the metric coefficients $g_{\mu\nu}$ vanish as well. This is the characteristic ‘repulson’ singularity,²⁹ whose resolution is the enhançon:³ the configuration with all the D5-branes at the origin is unphysical.

^dI would like to thank the authors of ref. 7 for correcting a factor of 2 in this and the subsequent equations.

Let us see this as in ref. 3, by starting with the constituent branes dispersed and trying to contract the distribution. For convenience we will regulate the UV also, as described above: we start with a ring of $N + M$ D3-branes at radius r_0 , in the orbifold limit $\theta_B = \frac{1}{2}$. Leave M anti-D5-branes at r_0 and contract the rest. When the inner ring is at radius r' we have

$$\theta_B = \begin{cases} \frac{1}{2}, & r > r_0 \\ \frac{1}{2} - 2gM \ln(r_0/r), & r_0 > r > r' \\ \frac{1}{2} - 2gM \ln(r_0/r'), & r' > r \end{cases} . \quad (22)$$

The inner ring is initially composed of $N + M$ D5-branes and N anti-D5-branes, but as it moves inward and $\theta_B(r')$ passes through integer values, the constituents reorganize as described in the previous section and the system cascades downwards, the number of branes of each type decreasing by M at each step.

For N a multiple of M , after $K = N/M$ steps there remain M D5-branes. When these reach the enhançon radius

$$r_e = r_0 \exp[-\pi(K + \frac{1}{2})/gM] \quad (23)$$

they become massless and the contraction must stop. At this point, all of the D3-brane charge of the inner ring has been stored in the three-form flux background. The density is nonnegative everywhere and there is no repulsion singularity. If N is not a multiple of M , there remain at the end some whole D3-branes.

In summary, the $\mathcal{N} = 2$ fractional brane system has several features in common with the $\mathcal{N} = 1$ system, but the singularity resolution and IR physics is the same as in the $\mathcal{N} = 2$ system considered in ref. 3. In the conclusions we will discuss some relations among these systems.

5. Open Questions

5.1. Exotic Physics

An important open question in this and related systems is the nature of the low energy physics, the physics of the enhançon ring.^e The final D5-branes sit at a radius where θ_B is an integer and so there are tensionless strings. Thus the effective low energy theory may be rather exotic. Note that in the present case there are additional low energy degrees of freedom on every ring where θ_B passes through an integer. As the constituents move inward and pass through a radius of integer θ_B , they rearrange, for example from the form (16) to (12), and the number of moduli drops by $2M$. The dimension of the $\mathcal{N} = 2$ moduli space cannot change, so these moduli must remain as degrees of freedom of the low energy theory on the ring. Thus the system exhibits an interesting analog of ‘spin-charge separation’: the D3-brane charge is spread throughout space in the form $F_3 \wedge H_3$, while the D3 moduli are localized at discrete radii.

^eSee also the talk by Clifford Johnson.

A related question concerns the dual at very large g . In the $\mathcal{N} = 4$ system, the effective description at $g \ll 1/N$ is a perturbative field theory, and at $g \gg N$ it is the dual field theory; in between is the supergravity description. In the present case, the description at $g \gg N$ is the field theory on wrapped NS5-branes, and it is not clear what this should be — there seems to be no simple orbifold interpretation. Note that in this limit the rings of integer θ_B are very closely spaced and so there are many exotic low energy degrees of freedom.

5.2. Related Solutions

It is interesting to take the T -dual on an ‘angular’ $U(1)$ at the \mathbf{Z}_2 singularity.^{30,3} This gives a pair of NS5-branes, extended in the 12345-directions and separated in the periodic 6-direction, with M D4-branes stretched between. The NS5-branes bend logarithmically away from one another and repeatedly intersect due to the periodicity — these intersections are at the radii of exotic low energy physics.

Now imagine the same brane configuration, rotated so that the separation and bending are in the 7-direction, while the 6-direction remains periodic. The shape of the branes is the same, though they no longer intersect. This T -dualizes back to a different solution: the 6-separation is T -dual to θ_B , while the 7-separation is T -dual to the volume of the S^2 . Thus one obtains a solution in which the S^2 does not remain at zero volume but expands logarithmically. This is the original context in which the enhancon was discussed,³ though the focus there was a yet-different T -dual form in which the volume of an entire K3 varies logarithmically.

Rotating one of the NS5-branes into the 12389 directions gives a T -dual to the $\mathcal{N} = 1$ KS solution.^{31,5} Thus the $\mathcal{N} = 1$ and 2 singularity resolutions can be continuously connected, even though one involves supergravity fields only and the other involves branes. Further, if this entire brane configuration is now rotated from the 6- into the 7-direction, one obtains a solution with vanishing NS-NS flux but a logarithmically growing S^3 — presumably the Chamseddine-Volkov solution,³² interpreted by Maldacena and Nuñez as dual to a confining gauge theory.⁶ The CV-MN and KS duals are different in the UV, the former being six-dimensional and the latter four-dimensional but with an unbounded gauge group; evidently the same number of degrees of freedom can be arranged in these two different ways.

The solutions in the 6-direction have constant dilaton and imaginary-self-dual three-form flux; the solutions in the 7-direction have real three-form flux with a related dilaton gradient. By rotating to an intermediate angle and taking the T -dual, one will obtain a solution of more general form.

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